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Active pole bending effect in pole support phase

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ABSTRACT

In pole vaulting, model analysis is one of the key methods to increase vaulting height. To date, the effects of athletes' motions during 'pole support phase' have been measured and modelled to improve and set new world records. The motions were extracted based on the context of pole bending interaction and parameters to improve vaulting height were investigated. However, due to experimental, mechanical, and sensing restrictions, ranges and interactions of the parameters were poorly addressed. To investigate further, a parameter space must be globally explored. Here, we show parameter sensitivities and interactive effects between initial velocity, pole length, bending amplitude and switching time. From the simulation studies, we found that active pole bending enabled successful pole vaulting with lower initial velocity and longer poles. Vaulting height had a local maximum point at a specific initial velocity and positive bending could control conditions to deliver the local maximum height. Positive bending controls the rising-up speed of the pole and contributes to the verticalisation of the vaulting angle. Negative bending increases the vaulting speed and contributes to the robustness of the vaulting angle. Our results demonstrate how these parameters affect the vaulting performances and suggest how athletes should activate their bodies.

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Pole vaulting; elasticity; kinetics; energy; simulation

Introduction

In pole vaulting, athletes skilfully convert their kinetic energy based on their initial horizontal velocity into the potential energy of vertical height with the use of long poles. The flexibility of the pole leads to efficient energy conversion (Arampatzis et al., 2004; Linthorne, 2000) and improved vaulting records (Dillman & Nelson, 1968). During the pole vaulting process, the motions of the athletes with a pole could be divided into different phases (Figure 1(a)). To improve vaulting height, the parameters for running up, planting a pole and taking off from the ground have been well studied, such as pole selection (Davis & Kukureka, 2012; Ekevad & Lundberg, 1997), approaching motion (Cassirame et al., 2019; Frère et al., 2009, 2017; Linthorne & Weetman, 2012),

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interactions during pole planting (Linthorne, 2018; Schade & Arampatzis, 2012), taking off angle (Angulo-Kinzler et al., 1994; Linthorne, 2000), etc.

Meanwhile, an athlete's motion during 'pole support phase', which is brought about by the large elasticity of the pole, has also drawn the attention of researchers (Arampatzis et al., 2004; Frère et al., 2012; McGinnis & Bergman, 1987; Morlier & Cid, 1996). In fact, an athlete's total energy when they crosses the bar can exceed 120% of initial energy at take-off (Frère et al., 2010). The additional energy is produced by the athlete's motion in the pole support phase (Arampatzis et al., 2004; Schade et al., 2000; Schade et al., 2006). The motions of athletes in this phase have been measured and analysed (Angulo-Kinzler et al., 1994; Ekevad & Lundberg, 1995; McGinnis & Bergman, 1987). Frère et al. measured electromyograms of the upper limb and showed that the local bending motion of the athletes to the pole brought additional potential energy to the systems and improved vaulting height (Frère et al., 2012). Schade et al. measured athletes' motions and analysed energy transition between athletes and poles (Arampatzis et al., 2004; Schade et al., 2000). Hubbard (1980) showed that the initial bending moment of the pole influenced the direction of the vaulting through numerical simulations. Nishikawa et al. (2013) examined the timing to start a swing-up motion and evaluated them with robot experiments (Nishikawa et al., 2013). Gudelj et al. (2013) examined the amplitude and timing of inversion motion from measurements of athletes. Fukushima et al. (2013) presented transitional buckling model and showed that the active bending motion of the athletes improved vaulting height by experiments tested with robots (Fukushima et al., 2014; Nishikawa et al., 2015). In their model, the athlete's motion is treated as bending moment on a pole and varies value of end support condition of Euler buckling model.

Although these studies investigated the effects of the active bending motions on the vaulting height and addressed potential parameters to improve vaulting height (initial velocity, pole length, pole stiffness, etc.), due to experimental, mechanical, and sensing restrictions of the objects, the ranges of the parameter values and the interaction of the parameters were poorly studied. To address the mechanism of how active bending motion improves vaulting height, a parameter space must be globally explored, and coupling effects among the parameters need to be investigated. In this paper, we globally vary the parameters that are the athlete's initial velocity, pole length, amplitude of active bending, and timing to switch bending directions to investigate the coupling effects of these parameters on the vaulting height. We hypothesise that the coupling effects of the parameters can explain the mechanism of how the active bending motion increases vaulting height.

Materials and methods

Active bending theory

Active bending theory describes the energy exchange between the athlete and the pole by a bending moment. Previous analyses of well-trained athletes' motions (Frère et al., 2012; McGinnis & Bergman, 1987; Morlier & Mesnard, 2007; Schade et al., 2000) showed that they bend poles as follows (Figure 1(b)):

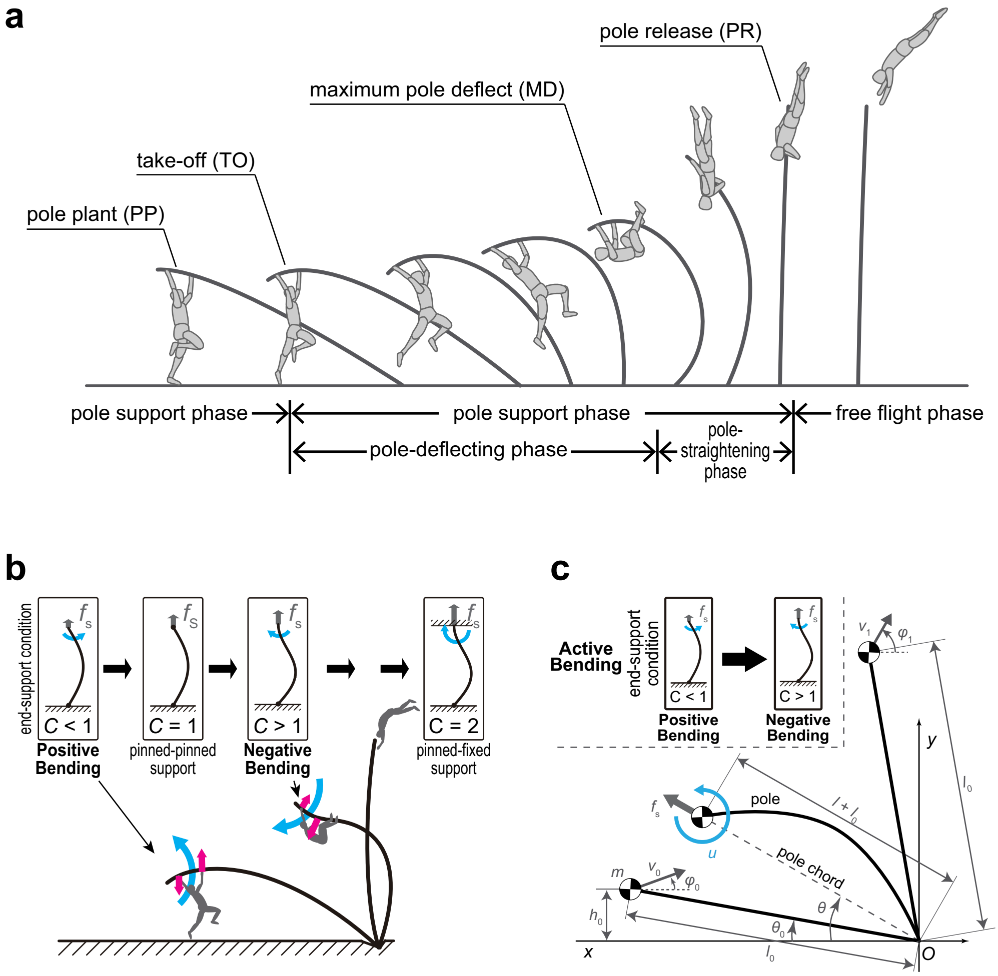

Figure 1. Numerical simulation model. (a) Phase diagram of pole vault. (b) Concept of transitional buckling model. Pink linear arrows show forces from arms. Blue curved arrows show input bending moments from arm forces. (c) Mathematical description of the transitional buckling model. Parameters correspond to Table 1. v_1 and ϕ_1 are respectively the point mass's velocity and angle when the pole completely re-straightens (displacement $l = 0$).

Phase 1: Pole-deflecting phase

By applying an upward force on the pole's lower hand grip, an athlete pushes back against their inertial force and prevents their body from drifting forward (Frère et al., 2010). This motion causes a bending moment to increase pole deflection (counterclockwise bending moment in Figure 1(b): positive bending). Intuitively, the pole then becomes more flexible and easier to deflect and therefore it stores more elastic potential energy.

Phase 2: Pole-straightening phase

By attracting the body to the pole and inverting it, the athlete exerts a bending moment to decrease pole deflection (clockwise bending moment in Figure 1(b), negative bending). Intuitively, the pole would then become stiffer and harder to deflect and therefore exerts a greater restoring force, which enables the athlete to achieve a higher vault. In fact, well-trained athletes straighten the pole faster than novice athletes (Morlier & Mesnard, 2007).

The athlete's motion generates a bending moment to transition the pole's imaginal stiffness. The former and latter bending actions are called 'positive bending' and 'negative bending' respectively in this study. We also call the series of these bending actions as 'active bending'.

Model construction: transitional buckling model

The pole vault movement was modelled in this work. A flexible pole's behaviour has been ordinarily modelled via multilink systems ekevad1995simulation (Ekevad & Lundberg, 1995; Nishikawa et al., 2015; Ohshima et al., 2010; Walker & Kirmser, 1973) or Euler buckling models (Hubbard, 1980; Linthorne, 2000; Liu et al., 2011). The latter was chosen to simplify the pole behaviour. In this Euler buckling model, reaction force from the pole is treated as a constant force, which coincides with an axis of the pole chord (buckling force). We modelled pole vault movement using this model and treated the athlete as a point mass (Figure 1(c)):

$$\frac{d}{dt}\begin{pmatrix}\theta\\ \dot{\theta}\\ l\\ \dot{l}\end{pmatrix} = \begin{pmatrix}\dot{\theta}\\ -\frac{1}{l+l_0}(2\dot{l}\dot{\theta}+g\cos\theta)\\ \dot{l}\\ (l+l_0)\dot{\theta}^2 - g\sin\theta\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 0\\ \frac{f_s}{m}\end{pmatrix} \tag{1}$$

$$f_s = C(u)\frac{\pi^2 EI}{l_0^2} \tag{2}$$

where C, E and I, respectively, represent the end-support condition coefficient, Young's modulus and area moment of inertia of the pole. The point mass has an initial horizontal velocity v_0. The pole exerts force f_s on the point mass. This force can be represented by the Euler buckling load. According to this model, f_s remains constant irrespective of pole deformation if the pole's end-support condition remains constant. Previous studies treated the end-support conditions of the pole as pinned supports at both top and bottom ends (Equation 2). Thus, C was usually set constant as $C = 1$ (Linthorne, 2000; Liu et al., 2011).

We instead represented active bending by the extended Euler buckling model. This model treated the pole's bottom end as a pinned support, but the top end is set as a variable support transitioning according to the input bending moment u (Figure 1(c)). By redefining C as variable $C = C(u)$, we proposed the transitional buckling model (TBM) to consider active bending (Equations 1 and 2).

Active bending and $C(u)$ are related as follows.

Phase 1 :$C(u) < 1$, **positive bending**

Intuitively, a value comparable to the spring constant is small. The pole exerts a small force f_s. Therefore, the point mass can deflect the pole with smaller force than with $C = 1$.

Phase 2: $C(u) > 1$, **negative bending**

Intuitively, a value comparable to the spring constant is large. The pole exerts a large force f_s. Therefore, the point mass can receive larger energy than with $C = 1$.

Our simple model has advantages in high amount of freedom for athletes' motion variety and it treats athletes' motions not as movements of individual body segments but as bending moment exerted by overall body movement. Therefore, it remains possibility for athletes to take various strategy for moving their body segments to exert certain bending moment.

Simulation experiments

To analyse the active bending effect, firstly, vaulting performances (vaulting height, vaulting angle, vaulting speed) with and without active bending ('active-bending' and 'non-actuation', respectively) were compared (experiment-1). Then, the each effect of positive and negative bending isolatedly was investigated (experiment-2). Furthermore, the best timing to switch bending direction was explored (experiment-3).

Common simulation setups

The simulations were performed by numerically solving the ordinary differential equations (Equations 1, 2) in time steps of 1 [ms] with the ODE45 solver in Matlab. Simulation parameters were defined by reference to actual athletes' data (Angulo-Kinzler et al., 1994; Ekevad & Lundberg, 1995) (Table 1). In the simulations, a point mass was suspended from a pole in a 2-D plane. As initial conditions, each state variable was given below (Figure 1(c)):

$$\begin{pmatrix} \theta \\ \dot{\theta} \\ l \\ \dot{l} \end{pmatrix} = \begin{pmatrix} \theta_0 \\ v_0 \arcsin(\theta_0 + \phi_0)/l_0 \\ 0 \\ -v_0 \arccos(\theta_0 + \phi_0) \end{pmatrix} \tag{3}$$

$$\theta_0 = \arcsin(l_0/h_0) \tag{4}$$

The point mass released the pole when the pole totally re-straightened ($l = 0$) and the time of the release was defined as t_{pr}. After t_{pr}, the mass point was then assumed to be in

Table 1. Simulation parameters.

Parameter	Unit	Description	Ex. 1	Ex. 2	Ex. 3	Resolution
θ	°	Elevation angle of the mass	Variable	←	←	–
x_m	m	x position of the mass	Variable	←	←	–
y_m	m	y position of the mass	Variable	←	←	–
$l + l_0$	m	Chord length of the pole	Variable	←	←	–
f_s	N	Exerted force from the pole	Variable	←	←	–
v_0	m/s	Initial velocity of the mass	6–10	←	←	0.08
l_0	m	Pole length	2–7	←	←	0.1
C_1	–	Amplitude of bending before t_{sw}	1.0, 0.8	0.6–1.4	0.8	0.2
C_2	–	Amplitude of bending after t_{sw}	1.0, 1.2	0.6–1.4	1.2	0.2
u_{sw}	m/s	Normalized time to switch bending direction	0	←	−10 − 10	0.4
t_{sw}	s	Time to switch bending direction	$t\|_{\dot{l}=u_{sw}}$	←	←	–
θ_0	°	Initial elevation angle of the mass	$\arcsin(l_0/h_0)$	←	←	–
m	kg	Mass of the mass point	80	←	←	–
h_0	m	Initial height of the mass	1.8	←	←	–
ϕ_0	°	Angle of initial velocity of the mass	20	←	←	–
g	m/s^2	Gravitational acceleration	9.8	←	←	–
E	GPa	Young's modulus of the pole	50	←	←	–
I	cm^4	Area moment of inertia of the pole	5	←	←	–

projectile motion with initial state at the pole release. To evaluate the vaulting performances, the following performance variables were defined:

$$\begin{cases} H_v & = & \max y_m|_{t \geq t_{pr}} \\ \phi_1 & = & \arctan(\dot{y}_m/\dot{x}_m)|_{t=t_{pr}} \\ v_1 & = & \sqrt{\dot{x}_m{}^2 + \dot{y}_m{}^2}\Big|_{t=t_{pr}} \end{cases} \tag{5}$$

here, x_m and y_m represented positions of the point mass in the plane. Vaulting height H_v was the maximum height of the point mass in the projectile motion. Vaulting angle ϕ_1 and vaulting speed v_1 were respectively the direction and norm of the velocity vector of the point mass at the pole release.

Parameters for 'non-actuation' and 'active-bending' were set as follows. In 'non-actuation', $C(u)$ was constant as $C = 1$:

$$C(u) = 1(t_{pp} \leq t \leq t_{pr}). \tag{6}$$

In 'active-bending', $C(u)$ was variable based on bending direction and its amplitude:

$$C(u) = \begin{cases} C_1 & (t_{pp} \leq t < t_{sw}) \\ C_2 & (t_{sw} \leq t \leq t_{pr}) \end{cases} \tag{7}$$

here, t_{pp}, t_{sw}, and t_{pr} were respectively the times when the pole was planted, the time when the bending direction changed, and the time the pole was released. To normalise t_{sw}, it was defined as follows:

$$u_{sw} = \frac{dl}{dt}\Big|_{t=t_{sw}}. \tag{8}$$

Here, u_{sw} was the pole deflection velocity. Therefore, $u_{sw} = 0$ was the time when the pole maximally deflected. During $u_{sw} < 0$ the pole was being deflected and during $u_{sw} > 0$ the pole was being straightened.

Under these conditions, we varied the initial speed and pole length 6–10 [m/s] in increments of 0.08 and 2–7 [m] in increments of 0.1, respectively (Table 1).

Experiment-1: comparison of 'non-actuation' and 'active-bending'

The vaulting performances between 'non-actuation' and 'active-bending' were compared. In 'active-bending', we set parameters as follows.

$$\begin{cases} C_1 & = & 0.8 \\ C_2 & = & 1.2 \end{cases} \tag{9}$$

$$u_{sw} = 0. \tag{10}$$

Experiment-2: effect of positive and negative bending

To investigate the effects of each positive and negative bending, we explored the vaulting performances while gradually changing C_1 and C_2 from 0.6 to 1.4 in increments of 0.2 independently (Table 1).

Experiment-3: timing to switch bending direction

To investigate the best timing for switching the bending direction, we explored switching timing, which could bring the highest vaulting height H_v, at each couple of initial velocity v_0 and pole length l_0. To normalise the switching time in different initial velocities v_0 and pole lengths l_0, we introduced normalised switching time u_{sw} being varied from -10 to 10 [m/s] (Equation 8, Table 1).

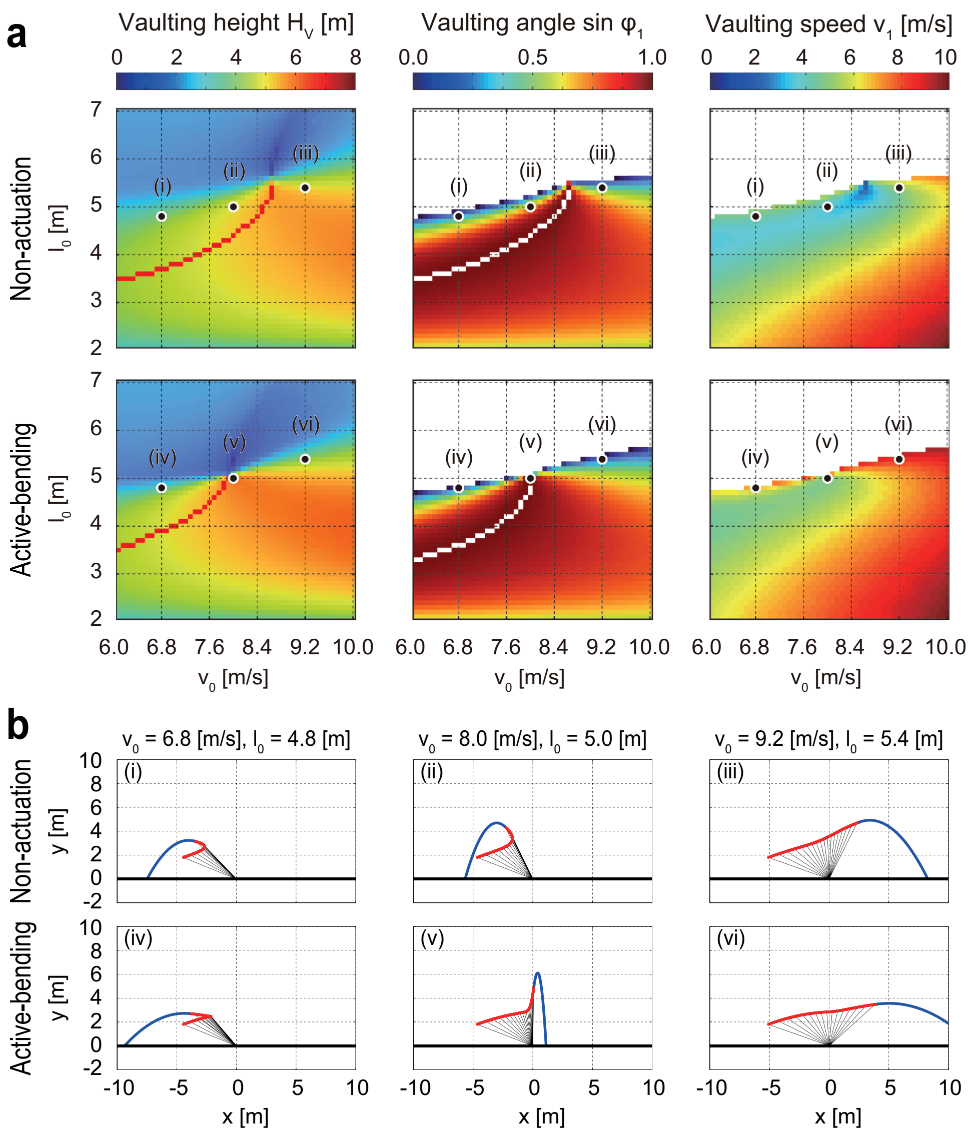

Figure 2. Results of simulation studies. (a) Map of vaulting performances (vaulting height, angle, speed) with varying initial velocity v_0 and pole length l_0 ('non-actuation' vs 'active-bending'). In 'non-actuation' $C_1 = C_2 = 1.0$. In 'active-bending' $C_1 = 0.8$, $C_2 = 1.2$. The red line on the vaulting height map showed combinations of initial speed and pole length which bring local maximum point (LMP). The white line on the vaulting angle map showed a peak of vaulting angle ($\sin\phi_1 \approx 1$). (b) Trajectories of point mass with each initial condition corresponding points (i)–(iv) in Figure 2(a). Red and blue lines, respectively, show trajectory of point mass during pole support and projectile motion phase. Black line shows pole chord, i.e., segment between upper and lower ends of pole, which is drawn every 100 [ms].

Results

Active bending brings successful vaulting (experiment-1)

Active bending enables successful vaulting at lower initial velocity

As a result, in 'active-bending', vaulting height H_v at each initial velocity v_0 was greater than in 'Non-actuation' (Figure 2(a): vaulting height). In 'active-bending', the area of high H_v expanded to lower initial velocity v_0 compared to 'non-actuation' (Figure 2(a): vaulting height).

Additionally, the map showed boundary lines (aqua colour in the colour map), which means the boundary of the successful vaulting, in both 'non-actuation' and 'active-bending' (Figure 2(a): vaulting height). Near the left boundary, the point mass rebounded backward, because the pole chord did not rise up adequately (Figure 2(a): vaulting angle, Figure 2(b): (i), (iv)). Near the right boundary, the point mass vaulted forward excessively because the pole chord fell over before the pole re-straightened (Figure 2(a): vaulting angle, Figure 2(b): (iii), (vi)). Along the ridge of $\sin\phi_1 = 1$ (the white lines in Figure 2(a): vaulting angle), the point mass vaulted vertically (Figure 2(b): (v)). Therefore, in left hand area of the ridge, the point mass vaulted backward and in the right hand area it vaulted forward respectively. In 'active-bending', the point mass could vault vertically or forward at lower initial velocity than in 'non-actuation' (Figure 2(b): (ii), (v), Figure 3(a)) at a pole length l_0. Therefore, with active bending, pole vaulting could be performed successfully at lower initial velocity, as shown in Figure 2.

Furthermore, the map showed that the initial velocity v_0 range where the point mass could vault vertically was limited to less than specific value of initial velocity v_{0opt}

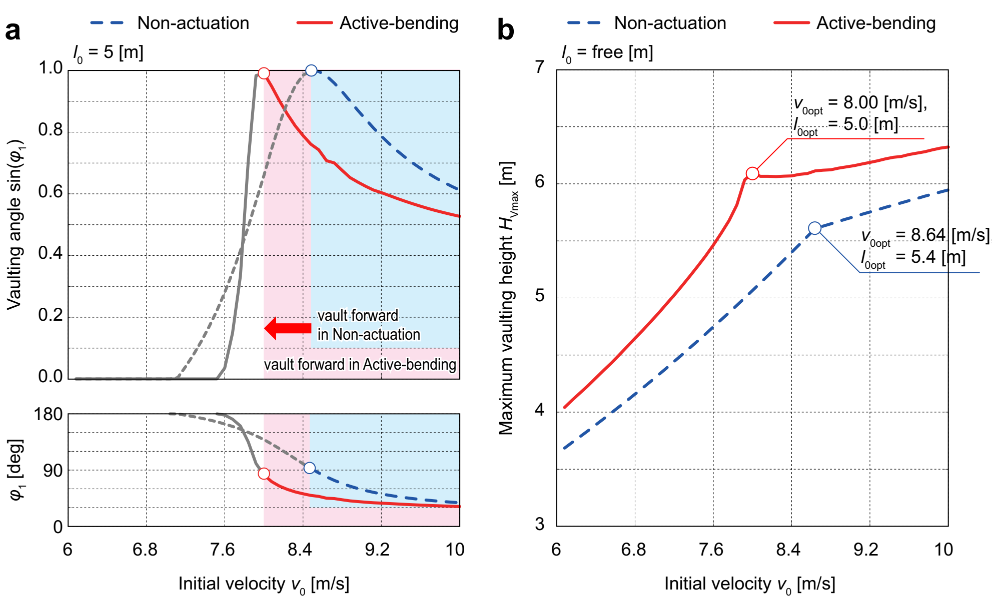

Figure 3. Vaulting performances in a cross-section of a pole length. (a) Cross section of the map (2) at $l_0 = 5$ [m]. Active bending allowed forward vaulting in lower initial velocity. (b) Maximum vaulting height H_{vMax} at each initial velocity v_0. H_{vMax} is defined as maximum vaulting height at each initial velocity v_0 with varying pole length from 2 to 7 m, assuming athlete can select adequate pole length. Active bending lowered required initial velocity for local maximum point (LMP).

($v_{0\mathrm{opt}} = 8.64$ [m/s] in 'non-actuation' and $v_{0\mathrm{opt}} = 8.0$ [m/s] in 'active-bending' in Figure 2(a): vaulting angle). At the combination of points $v_0 = v_{0\mathrm{opt}}$ and optimal pole length $l_{0\mathrm{opt}}$, vaulting height showed local maximum (Figure 3(b)). The local improvement was caused by specific parameters ($v_{0\mathrm{opt}}$ and $l_{0\mathrm{opt}}$) based on a certain pole's characteristics. Therefore, vaulting height could be improved at local maximum point (LMP) using an certain length of the pole. In the discussion section, the application of a LMP for coaching in pole vaulting is described in detail.

Active bending enables athletes to use longer poles

In pole vaulting, the maximum initial velocity v_0 of an athlete is generally limited. Therefore, the results were cut out at a certain initial velocity $v_0 = 8.0$ [m/s], which included a LMP, and compared.

Firstly, different trajectories of the point mass were compared. For a shorter pole, although vaulting heights were different, trajectories with 'non-actuation' and 'active-bending' showed little difference. In contrast, for a longer pole, trajectories showed a large difference. In 'non-actuation', the point mass rebounded backward because the pole re-straightened before it rose up vertically (Figure 2(b) (ii)). In 'active-bending', the point mass vaulted vertically because the pole re-straightened and rose up coordinately (Figure 2(b) (v)).

Additionally, vaulting angle $\sin\phi_1$ and vaulting speed v_1 at same initial velocity $v_0 = 8.0$ [m/s] were compared, which meant cross sections of the maps of Figure 2(a). In 'active-bending', the range of pole length l_0 by which the point mass vaulted vertically or forward was expanded to a longer pole ($l_0 = 4.9$ [m]) than in 'non-actuation' ($l_0 = 4.4$ [m]) (Figure 4(a)). Furthermore, vaulting speed v_1 was faster than in 'non-actuation' in the area of vertical or forward vaulting (Figure 4(b)). This was caused by a shifting optimal parameter values with active bending, which can bring the LMP condition.

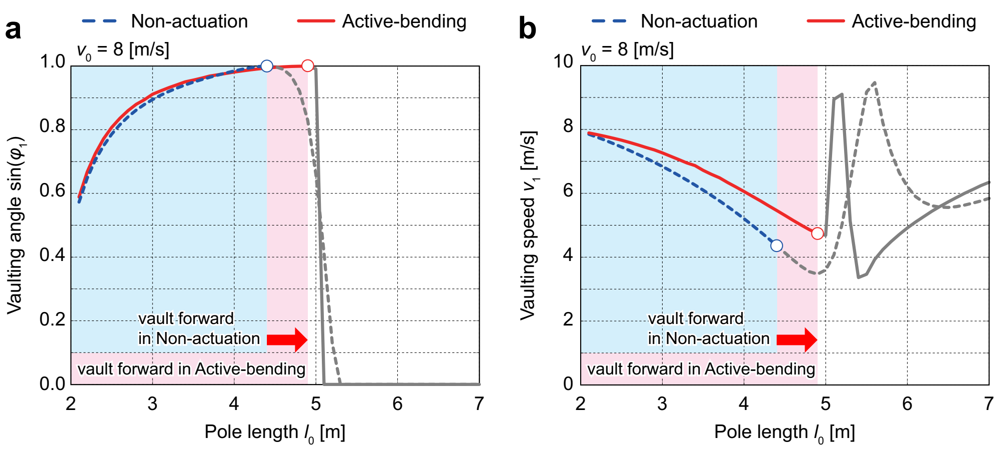

Figure 4. Vaulting performances in a cross-section of an initial velocity. Cross section of the map (Figure 2) at $v_0 = 8$ [m/s] in (a) vaulting angle and (b) vaulting speed. Blue coloured l_0 range indicates the pole length where the point mass vaulted vertically or forward in 'non-actuation'. Pink coloured one indicates the same in 'active-bending'. 'Active-bending' increased maximum pole length for successful vaulting.

Positive bending verticalise the pole angle and negative bending increases vaulting speed (experiment-2)

Upon increasing active bending amplitude (upper-leftward in Figure 5), the vaulting height increased. Additionally, when the point mass exerted bending moment inversely (negative and positive bending in pole deflecting and straightening phases respectively), the vaulting height decreased (lower-rightward in Figure 5).

Then, each effect of positive and negative bending amplitude was investigated. Upon increasing the positive bending amplitude (decrease of C_1, leftward in Figure 5), vaulting performances map's configuration changed and region where the point mass could vault high expanded to lower initial velocity and pole length range. Upon increasing the negative bending amplitude (increase of C_2, upward in Figure 5), map configuration generally remained unchanged, vaulting height increased though. Therefore, positive and negative bending had different effects on vaulting performances. We investigated these results in terms of vaulting angle ϕ_1 and vaulting speed v_1.

With increasing positive bending amplitude, LMP shifted to a lower initial velocity (Figure 5). This is due to the positive bending accelerated pole deflection, shortened the pole chord and thus, the moment of inertia from the pinned point (pole planting point) decreased. Therefore, pole rising speed $\dot{\theta}$ got larger, based on conservation of angular

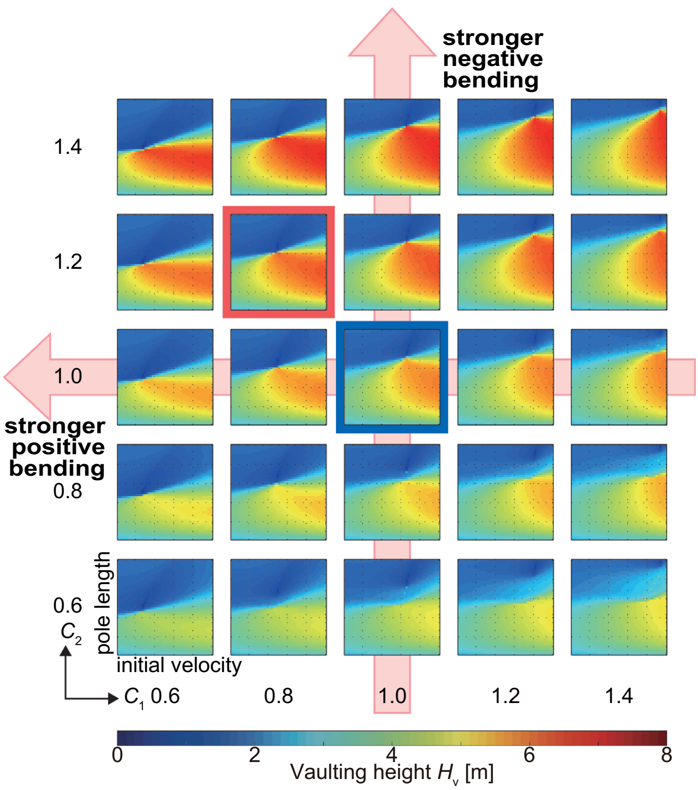

Figure 5. Vaulting height transition upon varying C_1 and C_2 from 0.6 to 1.4 in increments of 0.2. $C_1 = 1.0, C_2 = 1.0$ means 'Non-actuation' (blue coloured area) and $C_1 = 0.8, C_2 = 1.2$, 'Active-bending' (red coloured area) in Section 3.1. In each map, horizontal axis shows the initial velocity v_0 from 6 to 10 m/s and vertical axis shows the pole length l_0 from 2 to 7 m.

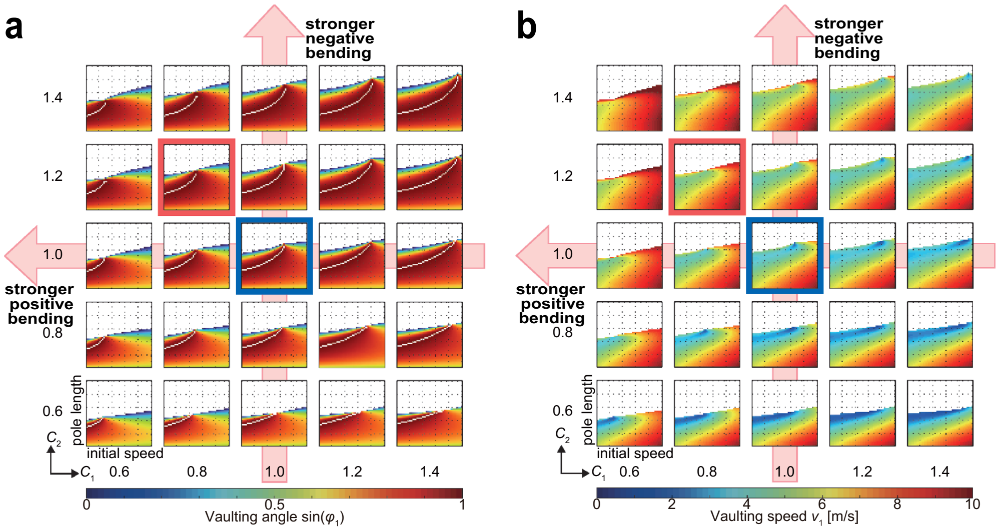

Figure 6. Vaulting angle $\sin\phi_1$ and vaulting speed v_1 transition upon varying C_1 and C_2 from 0.6 to 1.4 in increments of 0.2. $C_1 = 1.0, C_2 = 1.0$ means 'Non-actuation' (blue coloured area) and $C_1 = 0.8, C_2 = 1.2$, 'active-bending' (red coloured area) in Section 3.1. In each map, horizontal axis shows the initial velocity v_0 from 6 to 10 m/s and vertical axis shows the pole length l_0 from 2 to 7 m. The white line on the vaulting angle map showed a peak of vaulting angle ($\sin\phi_1 \approx 1$). The results showed positive bending controls the rising-up speed of the pole and contributes to the verticalisation of the vaulting angle and negative bending increases the vaulting speed and contributes to the robustness of the vaulting angle.

momentum and the point mass could vault vertically at a lower initial velocity (Figure 6(a)). Additionally, positive bending also increased vaulting speed v_1 (Figure 6(b)) because it greatly deflected the pole and increased its elastic energy; therefore, the pole exerted large energy on the point mass in its straightening phase.

Meanwhile, negative bending also increased vaulting speed v_1 (Figure 6(b)) because it gave energy to the pole and increased its straightening speed. Moreover, the parameter area where the point mass could vault vertically was expanded (Figure 6(a)). This is because quick pole straightening caused by negative bending completed pole re-straightening before the pole chord fell over forward, even it was over initial velocity for the pole length without the negative bending. Shortening of the time duration of the pole re-straightening brought a relatively larger parameter space (pole length and initial velocity) where the point mass was able to vault vertically; thus, it brought robustness for vertical vaulting.

In summary, positive bending contributed to verticalisation of the vaulting angle and achieved LMP at lower initial velocity. Negative bending contributed to the improvement of the vault speed and the robustness of the vault angle.

Early switching of the bending direction contributes robustness of the vaulting angle (experiment-3)

By selecting an appropriate timing to switch bending direction, successful vaulting area was expanded to higher initial velocity and longer pole (Figure 7: vaulting angle). From

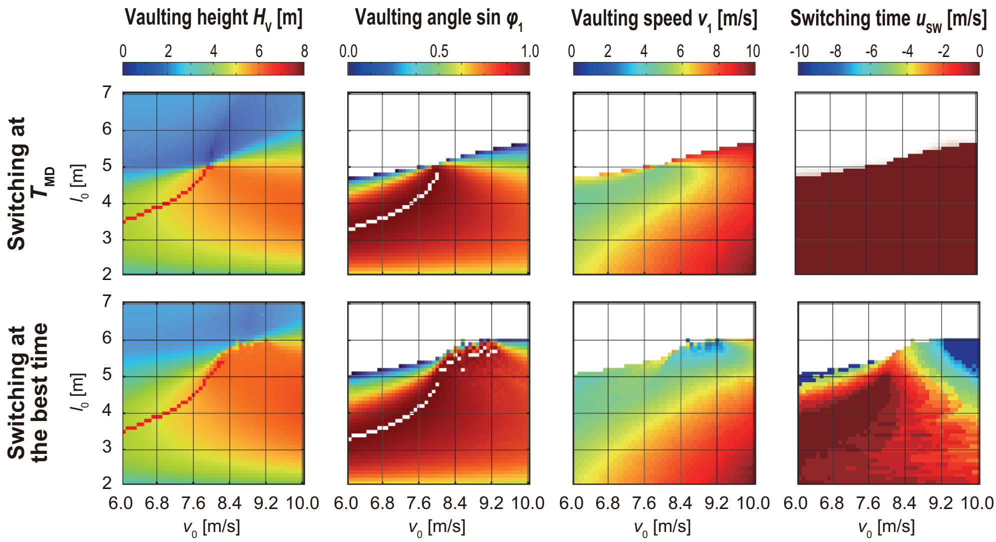

Figure 7. Effect of timing to switch bending direction. To normalise the switching time, normalised switching time u_{sw} was used (Equation 8). $u_{sw} < 0$ corresponds to pole chord shortening phase and $u_{sw} > 0$ corresponds to straightening phase. In each trial of 'switching at t_{md}', bending direction was switched when pole deflected maximally ($u_{sw} = 0$). In each trial of 'best switching time', switching time of bending direction was chosen to maximise vaulting height H_V. The maps of vaulting angle, vaulting speed, and the switching time were masked in unsuccessful vaulting area ($\sin\phi_1 < 0$). The white line on the vaulting angle map showed a peak of vaulting angle ($\sin\phi_1 \approx 1$). In the trials of 'switching at the best time, the switching time was varied $-10 \leq u_{sw} \leq 10$ [m/s], but the results of the best time were always $u_{sw} \leq 0$.

the context of the energy storage and consumption of the pole, the best timing to switch bending direction would be t_{md} when the pole maximally deflected ($u_{sw} = 0$ [m/s]). However, in high initial velocity range where the point mass vaulted forward, the best timing was before t_{md}. Although the vaulting speed v_1 decreased when switching the direction before t_{md} (Figure 7: vaulting speed), due to the increase in the vaulting angle $\sin\phi_1$, the vaulting height H_V increased in the high initial velocity range (Figure 7: vaulting height).

Discussion and implications

Mechanism to increase vaulting height by active bending motion

From the analysis of the coupling effects of the parameters, it was confirmed, from the pole bending aspect, that active bending allows athletes to use longer poles at lower initial velocity, and each contribution of positive and negative bending motion to the vaulting height was clarified. This result was also supported by literature of model studies with athlete's motion (Ekevad & Lundberg, 1997), in which athlete's active motion during pole support phase helped improve vaulting height. Positive bending controlled the rise-up speed of the pole and contributed to the verticalisation of the vaulting angle. This helps athletes shift LMP and vault vertically in lower initial velocity. This statement was also implied by literature based on model simulations (Ekevad & Lundberg, 1997; 1984) and measurements (Arampatzis et al., 2004), regardless of whether or not the athletes' active motions were treated as the active bending. Using our model, the statement can be explained by the shift of LMP condition, and applying high amplitude of active bending could be a solution to overcome the lack of initial velocity. On the other hand, negative bending increased the vaulting speed and contributed to the robustness of the vaulting angle. Athlete 'rock-back' motion, which increases CoG height, causes negative bending, but its effect on the bending moment has not been investigated well. Furthermore, certain combinations of initial velocity and pole length could bring a local maximal vault height. Although the simulation model cannot represent all aspects of the real pole vaulting phenomena, we believe that these findings would bring suggestions for vaulting strategies.

Pole vaulting strategy based on active bending theory

Here, we discuss how to apply the findings to actual pole vaulting. We assume two situations in which the vaulting is not optimal. The first is a case where a pole does not fully vertically rise due to a lack of initial velocity and the athlete bounces backward (the left-hand side of the white line in Figure 2(a), e.g., $v_0 = 8.0, l_0 = 5.0$). In this situation, the first consideration is to lower the gripping point (e.g., $l_0 = 5.0 \rightarrow 4.4$). However, this approach decreases the effective length of the pole and, even if the pole rises vertically, the vaulting height would decrease as well. Based on active bending theory, this shortage of the initial velocity could be compensated by an increase in the amplitude of the positive bending (Figure 2(a,b)). Therefore, it would be important to exert maximum bending torque to the pole in their capability to use a longer pole. The second is where a pole falls forward before its re-straightening and the athlete vaults forward excessively (the right-hand side of the white line in Figure 7, e.g., $v_0 = 7.6, l_0 = 3.0$). In this situation, a higher gripping point makes the moment of inertia larger and the pole rise-up time longer, and therefore the athlete can vault vertically (e.g., $l_0 = 3.0 \rightarrow 4.2$). However, in case the initial velocity exceeds the optimal initial velocity $v_{0\mathrm{opt}}$, no matter how high up the gripping point is moved, it will never be able to vault vertically (e.g., $v_0 = 9.2 > v_{0\mathrm{opt}}$). In this case, the athlete should switch to a negative vending motion earlier (Figure 7). Based on the results from Section 3.3, early switching and increasing amplitude of negative bending therefore had similar effects. The bending amplitude an athlete can exert is generally limited, but they would be able to control the vaulting angle by transit timing of their motion. This result of the early switching time is also supported in another multi-link pole vaulting model and robot experiments (Nishikawa et al., 2015).

Furthermore, after ensuring vertical vaulting using positive bending, athletes should increase their negative bending amplitude. High negative bending affords higher and robust vaulting (Section 3.2). Previously, the importance of positive bending in the pole deflecting phase has been emphasised by 'resisting motion' (Arampatzis et al., 2004; Frère et al., 2012; Hay, 1993; Tidow, 1989). However, the importance of negative bending in the pole straightening phase has not been emphasised much (Griner, 1984). Previous studies analysed the effect of negative bending on the exerted force using a pole deformation model without considering pole rotation around the pole planting box (Griner, 1984).

On the other hand, in our study, the pole planting model, which allows pole rotation, was used. As a result, it was found that this negative bending can largely enhance the vaulting ability without changing the global characteristics of high vaulting condition. In drills, athletes are instructed to take 'rock-back' posture in the pole straightening phase for fast inversion, because this posture has the smallest moment of inertia around the grip (Angulo-Kinzler et al., 1994). However, this minimises the negative bending amplitude instead. Based on the model studies, inversion motion with the stretched posture enhanced negative bending amplitude which can help to improve vaulting height (Section 3.3). On the other hand, this stretched inversion increases the moment of inertia and delays the completion of the inversion. For this, the earlier switching can help secure a longer time period for inversion with the stretched posture to adjust the completion of the inversion to the pole straightening timing. Therefore, active bending theory indicates that, with earlier switching, the athlete's inversion motion should be made in the most stretched posture (i.e., large body inertia), in which they can succeed the inversion.

Limitations of the model

Here, limitations of our model are discussed. To simplify the comparisons, in the model, some of aspects of real pole vaulting are omitted or condensed in the model. Since this is an extended model from the Euler's buckling model, the torque applied on a pole from athlete motion cannot be fully converted to the exerting force of the pole directly. On the other hand, this simplified model encapsulates the complex movements of the athletes and conveys them to the pole through a simple variable 'end-support condition'. It allows for a hierarchical simulation of the athlete's motion, the interaction between the athlete and the pole, and the pole vault as a whole system. The torques exerted by athletes were estimated from inverse kinematics (McGinnis & Bergman, 1987; Morlier & Mesnard, 2007) and simulations with the exerted torques were examined for robot development (Nishikawa et al., 2015). If the relationship between the torque and the end-support condition is formulated in this Transitional Buckling Model, it would be possible to calculate the active bending effect, including the complex body motion.

Secondly, in this model, the pole length was varied solely at a certain Young's modulus and an area moment of inertia. Defining 'pole stiffness' for pole vaulting, these parameters are not independent, and thus, the pole length parameter was varied solely in this study. This allowed us to find the specific combination of athlete's initial velocity and pole length values, which brought LMP. It is expected that if Young's modulus was varied instead, it would allow us to investigate more material aspects of poles. Also, other parameters, such as pole planting angle, take-off angle, distance of take-off etc., affect vaulting height. These parameters should be taken into account to the model for further investigation.

The other aspect is athletes' motion after a pole is fully re-straightened. In this model, it was assumed that fully pole straighten (PS) and pole release (PR) happens at exactly the same time and the point mass doesn't exert force during this period. For real-life pole vaulting, athletes are able to apply force and perform push-off action between PS and PR. This possibly has an effect on the vaulting height, the effect was not verified in measurements of athletes though (Frère et al., 2012). For the further elaborate simulation, this type of athlete motion should be implemented as well and be investigated.

Conclusion

We applied the active bending theory to pole vaulting simulations varying its parameters globally, and examined the coupling effects between the parameters on the vaulting performances. The results showed that active bending increased vaulting height, which had a local maximum point(LMP) at the combinations of certain values of initial velocity and pole characteristics, and that the parameter values that can lead to the LMP can be shifted by applying the active bending. In addition, we described optimal motions in terms of bending moments and presented the effects of positive and negative bending motions on vaulting performances. Positive bending controlled the rise-up speed of the pole and contributed to the verticalisation of the vaulting angle, which allows athletes to vault vertically at low initial velocity. Negative bending increased the vaulting speed and contributed to the robustness of the vaulting angle.

Furthermore, based on the active bending model with varying the initial speed, pole length, amplitude of active bending, and switch timing of bending direction, we confirmed the strategy of athletes to perform active bending using a longer pole and suggested the strategies of the earlier switch from positive to negative bending for controlling the vaulting angle and the stretched posture in the inversion motion for enhancing vaulting speed. We believe these suggestions would be able to help athletes to investigate their body motions during pole support phase and improve vaulting height.

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